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INFORMATION THEORY, QUARK CLUSTERS IN NUCLEI, AND PARTON DISTRIBUTIONS

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ABSTRACT

The procedure of maximization of information entropy can be used to improve our knowledge of parton distributions. This method has been applied in order to achieve the improved description of the nuclear effect in Y production due to gluon distribution modification in nuclei.

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1. INTRODUCTION

Information Theory was originally developed by Hartley [1], Nyquist [2], and Shannon [3] in order to establish a mathematical description of telecommunications and to understand how information may be lost upon transmission over noisy channels. Shannon, in particular, developed a complete formalism in which the concept of information is quantified and important theorems regarding its transmission are proven. The fundamental quantity that measures information is *information entropy*. Shannon has shown that the information entropy is the most suitable function of the probabilities for emission of signals by an ergodic source that measures the magnitude of the receiver's uncertainty on those signals. In this sense information entropy is the true measure of one's ignorance of their correct content. The larger the entropy the greater the uncertainty and, consequently, the information content. Therefore, maximizing the information entropy can lead to evaluation of the signal probability distribution under the constraints imposed by the telecommunications problem at hand. Application of this theory in physics could be versatile. All quantum phenomena, for example, are stochastic in nature and are described in terms of probability amplitudes. Then by appropriately defining the information entropy of the physical system under consideration and maximizing it under constraints imposed by theoretical assumptions or experimental data one can obtain the probability amplitudes that are most consistent with one's ignorance of the system. This method has been used by Plastino [4] to evaluate wave functions for various physical systems. It is a very powerful technique since it does not rely on any specific modeling but only on what is actually known about the system to derive best estimates for what is unknown.

In this article, we apply Information Theory to improve parton, specifically gluon, distributions in nuclei. These are suitable subjects to the method because they are probabilistic in character. The problem upon whose solution we wish to improve is that of quarkonium suppression in proton-nucleus collisions at very high energies. The production of charmonium states, most notably of the *J/Ψ* boson, as well as of bottomonium ones, especially the Υ resonance, has been observed in various experiments involving heavy nuclear targets to be lower than in hydrogen if the latter is multiplied by the mass number of the nucleus. At energies achieved at Fermi lab, *J/Ψ* and Υ suppression is very pronounced and exhibits a characteristic dependence on the momentum fraction of the target nucleon carried by the struck parton [5]. Many models have been developed to explain this behavior. For the purposes of demonstrating the Information Theory method, we consider a model that is based on the assumption that quarks in nuclei have a finite probability to conglomerate forming multi-quark color singlet states, usually called (multi)quark clusters [6]. The parton distributions in such clusters differ from those in single nucleons and generally are concentrated to lower momentum fractions of the partons as the cluster becomes larger. This model supplemented by final-state dissociation of the produced quarkonium has successfully described *J/Ψ* suppression in

Hadron-nucleus collisions and could possibly apply to heavy ion collisions, the latter being the topic of debate as they are important signals for the Quark-Gluon Plasma production. In this model very simple parton distributions have been used based on very general assumptions. However, the gluon distributions which play a crucial role in quarkonium production are relatively poorly known in nuclei. It turns out that within this model the gluon distributions that solve the problem of *J/Ψ* suppression are inadequate to describe Υ suppression from the same experiment. We shall use Information Theory to improve them in a manner that maintains their applicability to the *J/Ψ* data and, at the same time, enhances the agreement with the Υ data.

2. INFORMATION THEORY

Suppose an ergodic source of information, which can be anything from a telegraphic device to a quantum system, produces signals x from some available ensemble X with probability distribution $p(x)$. We define the information entropy of the source as [3]

$$S = -\int p(x,\alpha i)\ln p(x,\alpha i)\,\mathrm{d}x, \qquad (1)$$

Where the summation includes all instances of x in X and can indicate an integral if x is a continuous variable and αi are a group of fixed parameters in the function p. It is required that

$$\int p(x,\alpha i)\,\mathrm{d}x = 1. \qquad (2)$$

If the logarithm is binary then S is expressed in bits. Considered as a functional of the probability distribution, S is maximal when p is uniform; we know the most about a system when the signals it produces have no variability. To obtain the optimal function $p(x, \alpha i)$, i.e., to estimate the "best" set of parameters αi we impose the extremization condition

$$\frac{\partial S}{\partial \alpha_i} = 0, \qquad (3)$$

For all i. To ensure a maximum, second order derivatives must be looked at as well. In this work,we assume a certain functional form for p and simply want to determine its parameters. The procedure can be generalized to include cases in which we do not know the exact form of p but there are constraints based on data. The method of Lagrange multipliers can be used to evaluate distribution functions when a number of expectation values are known [4].

3. QUARKONIUM SUPPRESSION IN P-A COLLISIONS

3.1 Facts and Models

The observed suppression of the *J/Ψ*, $\Psi^{'}$ production cross section per unit mass number, *A*, in high energy hadron-nucleus [5, 7] and nucleus-nucleus [8] collisions exhibits a strong nuclear dependence. The hadron-nucleus data also show that the depletion increases with the longitudinal momentum of the quarkonium. These results have generated many theoretical studies. A variety of effects are thought

To contribute and numerous models have been suggested. These contributions may be grouped into six major categories: (1) Quarkonium scattering off parton and/or hadron co-movers [10, 11]; (2) Glauber inelastic scattering on the nucleons [12]; (3) Shadowing and EMC distortions of the nuclear parton distributions [13]; (4) Parton scattering before [14] and after [15] the hard process; (5) Parton energy loss in the initial and/or the final state [16] and, (6) Intrinsic charm in nucleons [11]. Color transparency may alter the Charmonium Glauber absorption [17]. In the case of heavy ion collisions contribution of an unconfined state (Quark- Gluon Plasma) has probably been very small in earlier experiments [5, 8] but is debated due to other data [18]. In our view, more than one contribution must be carefully balanced to explain the quarkonium suppression. Here the modifications of the initial-state parton distributions (for all values of the Bjorken variable, *x*) and the final state inelastic scattering (absorption) are considered.

The term "EMC effect", after the European Muon Collaboration, will signify any deviation from unity of the structure -function ratio of a bound nucleon to that of a free one at any value of the variable *x*, defined as the fraction of the nucleon momentum carried by the interacting parton.

3.2. Quark Clusters in Nuclei

The EMC effect has been studied in the framework of the expansion of a nuclear state on a complete basis of color-singlet states labeled by the number of their (3, 6, 9 or more) valence quarks. Such states, also referred to as multi-quark clusters, are formed when nucleons bound in a nucleus overlap so that they share their constituent partons.

The probabilities for multi-quark cluster formation can be estimated using nuclear wave functions [6] or can be found by fitting DIS data [19, 20]. The agreement with EMC and NMC [21] data is excellent down to $Q2 \approx 2$ GeV*2* and the fit strongly constrains the quark momentum distributions in clusters. (Momentum conservation in clusters fixes the *total* momentum fraction carried by the gluons.) Good description of the small enhancement above unity of the EMC ratio (ant shadowing) around $x = 0.1$ requires the inclusion of up to 12q clusters but the essential features of the data are accommodated by a truncation to the 6q term and use of an *effective* 6q cluster probability, *f*. The observed shadowing of the structure function, *F2*, in the nucleus for low *x* and the depletion for intermediate *x* combined with the QCD sum rules

Require the presence of ant shadowing of *F2* for other values of *x*. This ant shadowing, however, need not be restricted to the range $0.05 < x < 0.2$. In fact, this model predicts ant shadowing for $x > 0.8$ in agreement with the data of Ref. [21] on the slope of the Ca over deuterium structure-function ratio. In addition, the electron DIS data from SLAC confirm the excellent agreement of this model with the observed nuclear dependence for all $x>0.1$[22]. The model has been successfully applied to explain nuclear effects in Drell-Yan processes [23] and gives interesting predictions for these effects at RHIC energies [24]. Direct photon production in hadron nucleus collisions [25] has also been predicted to be altered by nuclear effects.

The probability *f* depends on *A* approximately logarithmically and is 0.040-0.052 for deuterium [25]. Very dense nuclei (*4*He) have *f* values larger than the logarithmic prediction. In the scaling limit, the Nq proton-like cluster parton momentum distributions are assumed to have standard forms,

$$V_N^{u,d}(x) = B_N^{u,d}\sqrt{x}(1-x)^{b_N^{u,d}}, b_N^d = b_N^u + 1 \tag{4}$$

$$S_N(x) = A_N(1-x)^{a_N}, A_N = x_N^S(1+a_N) \tag{5}$$

$$G_N(x) = C_N(1-x)^{c_N}, A_N = x_N^G(1+c_N) \tag{6}$$

The quantities x_N^S and x_N^G are the total momentum fractions carried by one sea quark species and the gluons, respectively. The exponents that best describe the NMC data are (b_3u, *a3*, b_6u, *a6*) = (3, 9, 10,11). For the gluon exponents the direct-γ data suggest (*c3*, *c6*) = (6, 10) [25]. The valence exponents approximately follow the dimensional counting rules. The ocean consists of three species of quarks (and their antiparticles) with the strange distribution being half as large as the up (or down) quark sea distribution. The gluon momentum fraction is taken 5 times larger than the ocean one. Isospin invariance relations connect the distributions that belong to the same isospin states reducing the number of independent parameters, e.g, $V_{proton}^u = V_{neutron}^d$. Kinematics forces the fraction of the cluster momentum carried by a parton in 6q clusters, *x(6)*, to be half as large as that in nucleons, *x(3)*. In this model, the QCD sum rules are explicitly obeyed. Further description of the quark-cluster model can be found in Refs. [24, 25].

3.3. Theoretical Cross Sections

Including 3q and 6q clusters, the doubly differential hadron-level cross section for quarkonium production is calculated from the order α_s^2parton-level ones (with both quark-antiquark annihilation ($q\bar{q}$) and gluon fusion (*gg*) included). The latter are convoluted with the appropriate sums of products of parton distributions $H_{q\bar{q}}^{(i)}(x_1, x_2^{(i)})$ and $H_{gg}^{(i)}(x_1, x_2^{(i)})$ (i=3,6) [26]. The indices 1 and 2 refer to the probe (moving in the +*z* direction in the laboratory) and the target, respectively.

The duality (in effect color evaporation) hypothesis is applied to integrate the differential cross section over m^2 with *m* ranging from twice the *c*(*b*) quark mass, 2*mc(b)*, to the open charm (bottom) threshold, 2*mD(B)*. Here m_D = 1.864 GeV and m_B = 5.278 GeV. The duality constant, *Fd*, is simply the portion of the total cross section (up to a given order) that corresponds to the quarkonium; it cancels in cross-section ratios. The resulting cross-section can be expressed as a function of the longitudinal momentum fraction carried by the produced quark-antiquark pair, $x_F = 2p_L/\sqrt{s}$. The higher order corrections are assumed to result in a multiplicative factor, *K*. The transverse momentum dependence due to higher order diagrams is, thus, integrated over and that due to the intrinsic transverse momentum of the partons is neglected. These considerations lead to the order α_s^2 equation

$$\frac{d\sigma^{(A)}}{dx_F} = K\, F_d \int_{4m_{c(b)}^2}^{4m_{D(B)}^2} dm^2 \sum_{i=3,6} J^{(i)}\left[H_{q\bar{q}}^{(i)}\hat{\sigma}_{q\bar{q}}(m^2) + H_{gg}^{(i)}\,\hat{\sigma}_{gg}(m^2)\,\right], \tag{7}$$

Where $\hat{\sigma}_{q\bar{q}}$ and $\hat{\sigma}_{gg}$ are the partonic-level cross sections for the two hard processes and *J(i)* are the Jacobians that transform *x1* and *x(i)* to x_F and *m2*.

In this way, the entire quarkonium yield is found. The various final states being unitary rearrangements of one another have up to this point the same dependence on EMC-type nuclear effects at the same $\sqrt{s}$. The perturbative*Q2* evolution of the parton distributions is omitted as it largely cancels in the calculation of cross section ratios. In addition, the

Q2 values that are relevant to the production of quarkonium states are inside the scaling region in which the parton distributions used in this work are valid.

3.4. Final State Absorption

After its production, the $c\bar{c}$ ($b\bar{b}$) system propagates in the nuclear medium developing into a quarkonium state which then decays into the observed lepton pairs. During this stage, the system may be inelastically scattered by 3q and 6q clusters. It is reasonable to assume that the scattering occurs with clusters bound in the nucleus since the break-up time of the latter exceeds the time the pair needs to traverse the nuclear radius. The *J/Ψ* absorption has been measured to be $\sigma^{(3)}_{abs} = 3.5$ mbarn/nucleon [27]. This number has been extracted in the kinematic range in which parton distribution modifications are negligibly small ($x_2 \approx 0.22$, the point at which the EMC ratio crosses the unit line) and since it is the average absorption cross section over the path traversed in the nucleus, color transparency effects are already in it. The Ψ´ is attenuated a little more than the *J/Ψ* due to its larger radius. It is also assumed that the cross section for bottomonium (Υ states) absorption is of the same order as that of charmonium but smaller (≈3.0 mbarn/nucleon) due to its more compact size.

The cross- section on a 6q cluster is 2*3/2* times larger than that on a nucleon (bag model estimate) but the density of scattering centers in the medium is reduced

when $f \neq 0$. Then

$$\left(\rho\sigma_{abs}\right) = \sigma^{(3)}_{abs}\rho_A\left[(1-f) + 2^{\frac{3}{2}}f\right]/(1+f), \tag{8}$$

Where ρ_A is the number density of the nucleus *A* taken as constant within the nuclear volume. The average path length the $c\bar{c}$ ($b\bar{b}$) pair travels in an

approximately spherical nucleus of radius r_A estimated by means of a simple geometrical (not eikonal) calculation is $L_A \approx 2r_A/\pi$. The experimentally measured nuclear RMS radii [28] are employed to compute ρ_A and L_A. The cross-section in Eq.(7) is then attenuated by the factor

$$P_A = \exp[-(\rho\sigma_{abs})L_A]. \tag{9}$$

4. CHARMONIUM SUPPRESSION

The nuclear dependence is extracted by taking the ratio, R_A, of the *J/Ψ* production cross section per unit *A* in collisions of a hadron, proton in this case, with a heavy nucleus to that in collisions with a light one and examining its x_F dependence. Since the *K*-factor may also depend on x_F at first we examine the ration of the experimental to the order α^2_s theoretical cross section. In Figure 1 we present this ratio for Cu and Be using a representative set of parameters for our model and the data of Ref. [7]. *Fd* is the same in both cases and cancels in the ratio. Clearly, the *K*-factor exhibits a strong x_F dependence but, most importantly for our purposes, is the same for the two nuclei. This implies that it will cancel in ratios of two theoretical or experimental cross sections. The origin of this factor must, thus, lie in processes that do not depend on the size and intrinsic properties of the nucleus (the absorption part is included in the results of Figure 1). The x_F dependence of *K* in Figure 1 agrees with that of the ratio of the "diffractive" to hard cross sections in Ref. [29]. Issues related to the relative suppression of various charmonium states are outside the scope of this article.

The results for R_A are confronted with the data of Ref. [5] in Figure 2. The dotted lines in Figure 2 represent the prediction of the full model including 6q clusters and final state absorption with the lowest (highest) value of f for the heavy (light) nucleus and the largest values of the 6q ocean and gluon exponents, ($a6$, $c6$) = (12, 11); the solid ones correspond to the opposite f combination and ($a6$, $c6$) = (10, 9). In order to make the influence of the initial and final state contributions to R_A clear in Figure 2(d) the results without final state absorption are shown (short and long dash curves with the same connotation as the dotted and solid ones, respectively) as well as the result with only final state absorption, $f = 0$ (dot-dash line). It is evident that the predictions of the full model are in agreement with the data. We note in passing that for large negative x_F this model predicts ant shadowing of J/Ψ production in p-A collisions because in this region large x_2 values for the gluon distribution ratio are accessed.

5. BOTTOMONIUM SUPPRESSION

Using the model we described earlier we can also calculate the suppression ratio for the Y states and compare the results with the data of Ref. [9]. Specifically, we calculate the exponent α defined by means of the equation

$$\frac{d\sigma^{(A)}}{dx_2} = A^{\alpha} \frac{d\sigma^{(d)}}{dx_2} \tag{10}$$

where the superscripts refer to large nuclei (A) or deuterium (d) targets and $x_2 = x_2^{(3)}$.At this point, it is instructive to observe that gluon fusion dominates the charmonium production process and is very important for bottomonium production as well. Therefore, the ratio of cross sections to a large extent reflects the ratio of gluon distributions, $R_G^{(A)}$. It is not hard to see that with the given definition of gluon distributions $R_G^{(A)}$ monotonically increases with x_2. As shown in Figure 3, however, the data of Ref. [9] contradict this theoretical prediction. At $x_2 \approx 0.15$ there is a wide "bump" and the ratio starts decreasing at higher x_2. The reason that this behavior becomes more apparent in the case of Y production is the fact that the Y is much more massive than the J/Ψ. For given center of mass energy, a particular quarkonium momentum, x_F, probes a larger x_2 value in the Y case. The gluon distributions being the most relevant and the least known among all the partons should be the first candidates for improvement.

5.1. Improved Gluon Distributions

At this point, we turn to Information Theory. The total momentum fractions carried by the partons in an Nq cluster,

$$z_N^{(a)} = \int_0^1 dx\, F_N^{(a)}(x) \tag{11}$$

where $\boldsymbol{a}$ designates the type of parton and $F_N^{(a)}$ is its momentum distribution. The fractions must always add up to unity. Consequently, the functions $F_N^{(a)}$satisfy the condition required in order to define an information entropy,

$$S_N = -\sum_a \int_0^1 dx\; F_N^{(a)}(x) \ln F_N^{(a)}. \tag{12}$$

We shall maintain the quark distributions as defined in the previous section and modify the gluon distributions under the constraint that the total momentum fraction carried by gluons in each type of cluster is fixed and equal to that of the unmodified distributions, i.e., $z_3^{(g)} = 0.57$ for nucleons and

$z_6^{(g)} = 0.60$ for 6q clusters.

The trend of the Y data suggests that the simplest possible modification to the gluon distributions is an alteration of the linear term in x. We will, then, define corrected momentum distributions for the gluons in nucleons and 6q clusters as

$$G_N(x) = C_N(1-x)^{c_N} + C_N^{\prime}\, x. \tag{13}$$

For each N we, now, have two unknown parameters, C_N and $C_N^{\prime}$. Momentum conservation, i.e., the fractions $z_N^{(g)}$ are constant, imposes the condition

$$C_N^{\prime} = 2z_N^{(g)} - 2C_N/(c_N+1) \tag{14}$$

Where the exponents c_N are kept fixed, $c_3 = 6$ and $c_6 = 10$. Then we evaluate

C_N from the requirement

$$\partial S_N/\partial C_N = 0. \tag{15}$$

The maximization procedure yields the following numbers: (C_3, C'_3) = (1.163, 0.812) and (C_6, C'_6) = (1.411, 0.987). We can compare these numbers with the uncorrected ones: (C_3,C'_3)*uncorr* = (4.130, 0.0) and (C_6,C'_6) *uncorr =3 6*(6.624, 0.0). We have changed the shape of the function without affecting its integral, the total gluon momentum. The behavior of the new functions differs from that of the old ones mostly in the large x region for which on the other hand we have little experimental data. We note that the ocean distributions could not accommodate such type of alteration because deeply inelastic scattering imposes a constraint on the ratio of neutron to proton structure functions, tends to 1/4 as $x \rightarrow 1$. Using the corrected distributions we can recalculate the exponent α and compare the results with the data. With a final state absorption cross section of 3 mbarn/nucleon and the new distributions the agreement with the data is considerably improved as shown in Figure 3 in which the upper curve corresponds to the uncorrected model, the lower curve to the corrected model using only gluon fusion contributions and the middle curve to the corrected model including gluon fusion and quark annihilation. It must be pointed out that this is not the only model that gives reasonable description of the relative *J/Ψ* to Y suppression data in p-A collisions. The authors of Ref. [30] attribute the smaller suppression of Y to the *Q2* ≈ *m2* evolution of the distribution functions, where *m* is the mass of the resonance. Indeed, as discussed in Ref. [26] the evolution of the ocean (and consequently the gluon) distributions leads to smaller shadowing as *Q2* decreases. Our model neglects the *Q2* evolution relying on the observation [26] that the masses of the quarkonium resonances are already in the scaling region and attributes the reduced Y suppression to its smaller absorption cross-section. It is conceivable that both effects may, in fact, contribute to this phenomenon. We would mostly like to emphasize that it is the shape of the suppression curve that needed to be improved to account for the Y data.

It is worthwhile noting that due to the fact that Y production probes a different kinematic regime from *J/Ψ* the correction on the gluon distributions has only a small effect on the *J/Ψ* suppression curves. The agreement with the charmonium data is still good as it can be observed in Figure 4, although it slightly deteriorates at small x_F (large x_2). In addition, the corrected model does not exhibit ant shadowing of *J/Ψ* production for negative x_F a feature that would be in contradiction with the data [31]. Data on bottomonium and charmonium give complementary information on the gluon distributions in nuclei. In addition, the Y is a really good probe of the initial state in which it is produced due to low absorption cross-section.

5.2. Relativistic Heavy Ion Collisions

We can use this model to make predictions for the *J/Ψ* and Y suppression at the Relativistic Heavy Ion Collider (RHIC) with $\sqrt{s}$ = 200 GeV/nucleon.

The calculation proceeds along the same lines as for p-A collisions but now there is the additional possibility of 6q-6q cluster collisions. The nuclear effect is, thus, more pronounced. The cross section for collisions of a nucleus A with a nucleus B is [26]

$$\frac{d\sigma^{(AB)}}{dx_F} = K\ \text{橦}_d \int_{4m^2_{\text{敌}(b)}}^{4m^2_{D(B)}} dm^2 \sum_{i=3,6} \sum_{j=3,6} J^{(i,j)} \left[H^{(i,j)}_{q\bar{q}}\ \hat{\sigma}_{q\bar{q}} + H^{(i,j)}_{gg}\ \hat{\sigma}_{gg}\right] \quad (16)$$

where, $J^{(i,j)} = x_1^{(i)}\, x_2^{(j)} / m^2 \left(x_1^{(i)} + x_2^{(j)}\right)$, i and j represent the type of colliding cluster, $\sqrt{s}$ is the nucleon-nucleon CM energy and $H^{(i,j)}_{q\bar{q},gg}$ are functions of parton distributions appropriate for A-B collisions.

In Figure 5 we show theoretical results obtained with the uncorrected (curves marked by (d)) [26] and the corrected (curves marked by (c)) models. The curves exhibiting less suppression are for the Y. The corrected model leads to larger suppression. The large x_F behavior is now much more distinct. It can be understood if we realize that in symmetric heavy ion collisions as $x_F \rightarrow 1$ the large x region for the positively moving nucleus is probed. Therefore, in the uncorrected model the ratio increases with x_F reflecting the increase in the gluon distribution ratio and exhibits anti-shadowing while in the corrected one it flattens out and remains below unity. In Ref. [32] data produced by the PHENIX Collaboration were presented. The value of the Y suppression ratio for Au-Au minimum-bias events versus p-p collisions at $\sqrt{s}$ =200 GeV is found to be 0.625 ± 0.200. This falls into the theoretically predicted band.

6. CONCLUSIONS

We have applied Information Theory to improve the gluon momentum distribution functions in nuclei, including the “EMC effect”. The main idea is that by defining an information entropy, *S*, for those functions whose total integral is unity we can evaluate their parameters by maximizing *S* with respect to them. In other words, we assume that the best choice of parameters is the one that is consistent with maximal ignorance under the constraint of momentum conservation. A quark-cluster model for the “EMC effect” has been used to establish a good agreement with the data on *J/Ψ* suppression in p-A collisions but proved inadequate to describe Y suppression. Then Information Theory provided us with a tool to improve the model with significant success. The gluon distributions have been corrected for their behavior at large *x* and an overall agreement with the differential cross sections for quarkonium suppression was achieved.

An interesting aspect of this method is that it does not rely on any specific microscopic theory which in turn should be investigated in detail but uses only very general notions. The solution that is consistent with the assumption of maximal ignorance, quantified by the information entropy, seems to be an optimal one. This method can be used to improve the parameters of more detailed and realistic parton distributions in nucleons and nuclei under constraints imposed by experimental data.

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APPENDICES

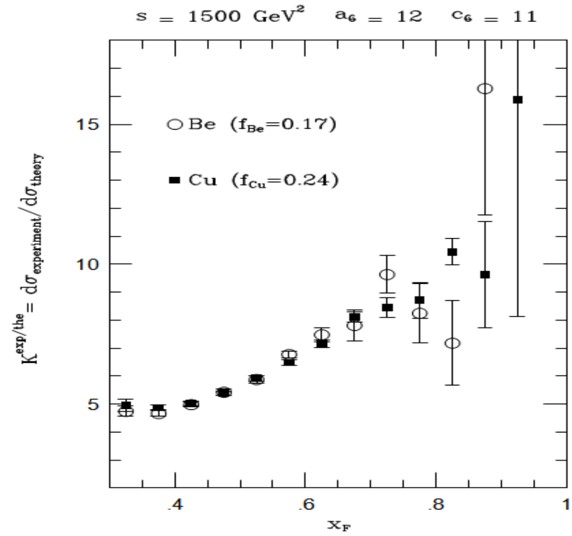

Figure 1: The Ratio of the Experimental to the Theoretical Cross Section for *J/Ψ* Production in Collisions of Protons with Cu and Be Nuclei. The Theoretical Cross Section Includes Cluster Contributions and Final State Absorption. The Experimental Cross Section is From Ref. [7] with Statistical and Systematic Errors Added in Quadrature. The Uncertainty in the Theoretical Cross Section is Not included

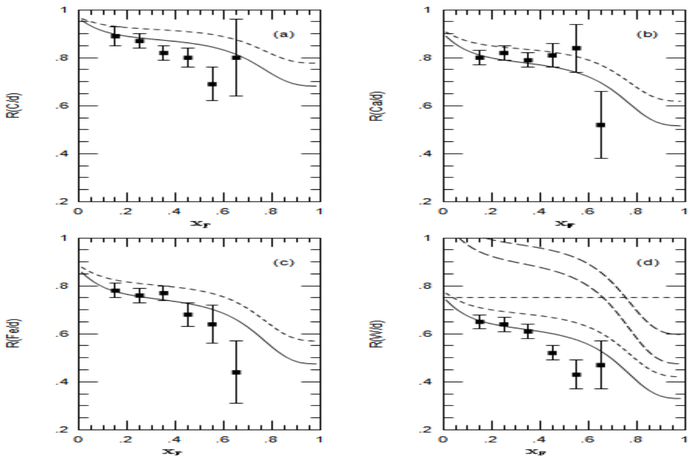

Figure 2: C (a), Ca (b), Fe (c), and W (d) to Deuterium *J/Ψ* Production Ratio Versus x_F. The 6q Probabilities are Evaluated using $f = k \ln A$ with 0.040-0.052 for Deuterium. The 6q Ocean and Gluon Exponents are 10, 9, Respectively for the Solid Lines and 12, 11 for the Dotted Ones. Panel (d) Also Shows the Results for no Absorption (Short and Long Dash Curves) and with Absorption only (Dot-Dash Line). The Data are from Ref. [5] at $\sqrt{s}$ =38.7 GeV. The Errors are Statistical

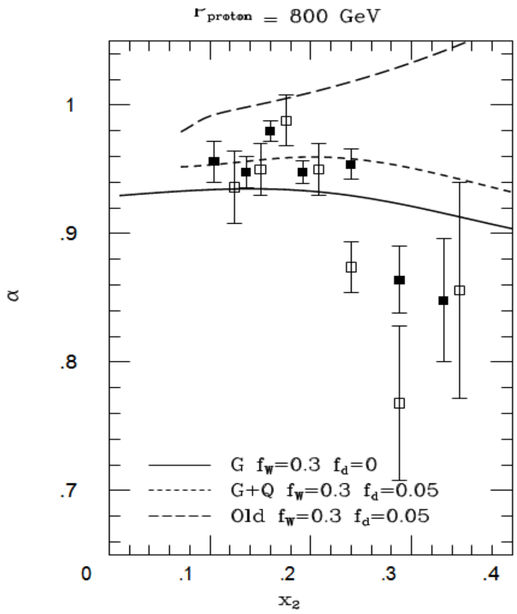

Figure 3: Exponent of the W to Deuterium Y-production Ratio Versus x_2. The Chosen 6q Probabilities are Shown in the Plot. The Data Points are from Ref. [9]. The Solid Squares are for the 1*S* State and the Open Ones for 2*S* + 3*S*. The Upper Curve Corresponds to the Uncorrected Model. The Lower Curve Corresponds to the Corrected Model using Only Gluon Fusion Contributions. The Middle Curve Corresponds to the Full Corrected Model Including Quark Annihilation

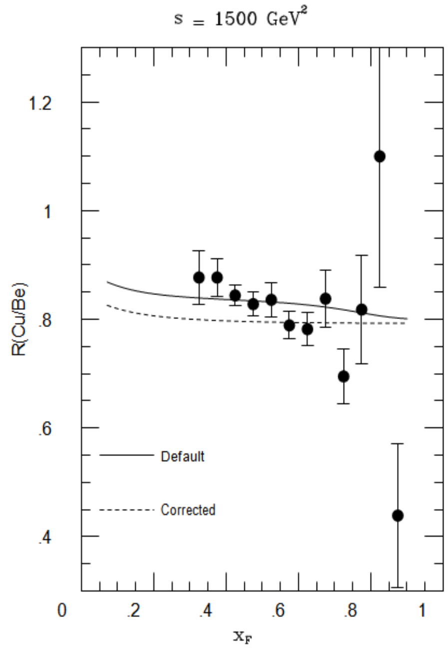

Figure 4: The Ratio of J/Ψ Production Cross Section per Nucleon on Cu to Be Targets with a Proton Beam at $\sqrt{s}$ =38.72 GeV. The Solid Curve is for the Uncorrected Gluon Distributions and the Dash Curve for the Modified Ones. The Exponents Used are (c_3, c_6) = (6, 10) for the Gluons and (a_3, a_6) = (9, 11) for the Ocean. The Effective 6q Cluster Probabilities are 0.30 for Cu and 0.16 for Be. The Data Points are from Ref. [7]

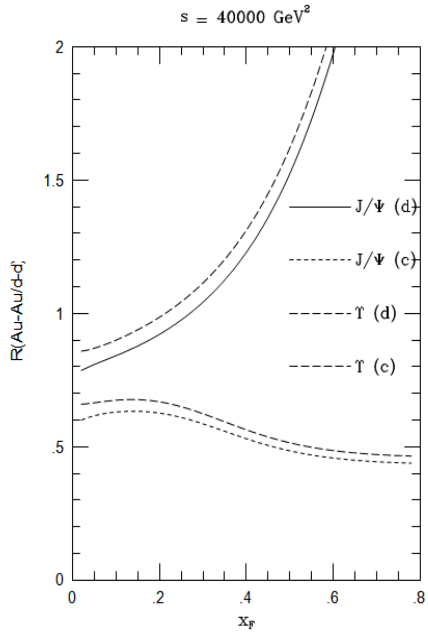

Figure 5: The Ratios of *J/Ψ* and Υ Production Au-Au Cross Sections Per Nucleon Over Deuterium Ones at RHIC Energies. The Curves Marked by (d) are for the Uncorrected Model and Those Marked by (c) for the Corrected one. The Effective 6q Probabilities are 0.40 for Au and 0.05 for Deuterium. The used Exponents are (c_3, c_6) = (6, 10) for the Gluons and (a_3, a_6) = (9, 11) for the Ocean. The Absorption Cross Sections are 3.5 Mbarn/Nucleon for the *J/Ψ* and 3.0 mbarn/nucleon for the